MATH 111*i*

WORKSHEET 3

Name: Solutions

Work in groups to answer as many problems as you can. Ask questions if you get stuck.

1. Factor out the greatest common factor of each of the following polynomials.

(a) $5a - 20$

Answer: $5(a-4)$

(b) $30x^3 + 15x^4$

Answer: $15x^3(2+x)$

(c) $-2x^3 + 16x$

Answer: $-2x(x^2-8)$

(d) $-3b + 12$

Answer: $-3(b-4)$

(e) $12x^3 + 18x$

Answer: $6x(2x^2+3)$

(f) $6y^4 - 15y^3$

Answer: $3y^3(2y-5)$

(g) $2x^4 + 4x^3 - 14x^2$

Answer: $2x^2(x^2+4x-7)$

(h) $y(y-6) + 9(y-6)$

Answer: $(y-6)(y+9)$

(i) $(z+2)^2 - 5(z+2)$

Answer: $(z+2)(z-3)$

2. Factor each expression.

(a) $x^2 + 2x - 3$

Answer: $(x+3)(x-1)$

(b) $x^2 + 2x - 15$

Answer: $(x+5)(x-3)$

(c) $y^2 - 8y + 15$

Answer: $(y-5)(y-3)$

(d) $2x^2 - 5x - 7$

Answer: $(2x-7)(x+1)$

(e) $5x^2 - 7x - 6$

Answer: $(5x+3)(x-2)$

(f) $9x^2 - 36x - 45$

Answer: $9(x-5)(x+1)$

(g) $x^2 - 6x + 5$

Answer: $(x-2)(x-3)$

(h) $x^2 - 14x + 48$

Answer: $(x-6)(x-8)$

(i) $z^2 + 6z - 16$

Answer: $(z+8)(z-2)$

(j) $3x^2 - 16x + 5$

Answer: $(3x-1)(x-5)$

(k) $2x^2 + 7x - 4$

Answer: $(2x-1)(x+4)$

(l) $8x^2 + 10x + 3$

Answer: $(2x+1)(4x+3)$

3. Using the formula $A^2 - B^2 = (A + B)(A - B)$ to factor the following.

(a) $x^2 - 36$

Answer: $(x-6)(x+6)$

(b) $9a^2 - 16$

Answer: $(3a-4)(3a+4)$

(c) $49 - 4y^2$

Answer: $(7-2y)(7+2y)$

(d) $(x + 3)^2 - 4$

Answer: $(x+1)(x+5)$

(e) $(a + b)^2 - (a - b)^2$

Answer: $(2b)(2a)$
$= 4ab$

(f) $y^2 - 100$

Answer: $(y-10)(y+10)$

(g) $4x^2 - 25$

Answer: $(2x-5)(2x+5)$

(h) $4t^2 - 9s^2$

Answer: $(2t-3s)(2t+3s)$

(i) $4(2x + 1)^2 - 9$

Answer: $(4x-1)(4x+5)$

(j) $\left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2$

Answer: $(2)(\frac{2}{x})$
$= 4/x.$

Cont.

4. Factor each of the following perfect squares.

(a) $x^2 + 12x + 36$

Answer: $(x+6)^2$

(b) $t^2 - 6t + 9$

Answer: $(t-3)^2$

(c) $16z^2 - 24z + 9$

Answer: $(4z-3)^2$

(d) $4w^2 + 4wy + y^2$

Answer: $(2w+y)^2$

(e) $y^2 + 10y + 25$

Answer: $(y+5)^2$

(f) $t^2 - 10t + 25$

Answer: $(t-5)^2$

(g) $25u^2 - 10u + 1$

Answer: $(5u-1)^2$

(h) $r^2 - 6rs + 9s^2$

Answer: $(r-3s)^2$

5. Factor the following polynomials by grouping.

(a) $x^3 + 4x^2 + x + 4$

$= (x^3 + 4x^2) + (x+4)$

$= x^2(x+4) + (x+4)$

Answer: $(x^2+1)(x+4)$

(b) $2r^3 + r^2 - 6r - 3$

$= (2r^3 + r^2) - (6r+3)$

$= r^2(2r+1) - 3(2r+1)$

Answer: $(r^2-3)(2r+1)$

(c) $x^3 + x^2 + x + 1$

$= (x^3 + x^2) + (x+1)$

$= x^2(x+1) + (x+1)$

Answer: $(x^2+1)(x+1)$

(d) $y^3 - 3y^2 - 4y + 12$

$= (y^3 - 3y^2) - (4y - 12)$

$= y^2(y-3) - 4(y-3)$

$= (y^2-4)(y-3)$

Answer: $(y+2)(y-2)(y-3)$

(e) $2t^3 + 4t^2 + t + 2$

$= (2t^3 + 4t^2) + (t+2)$

$= 2t^2(t+2) + (t+2)$

Answer: $(2t^2+1)(t+2)$

(f) $3x^3 - x^2 + 6x - 2$

$= (3x^3 - x^2) + (6x-2)$

$= x^2(3x-1) + 2(3x-1)$

Answer: $(x^2+2)(3x-1)$

Cont.

(g) $-9u^3 - 3u^2 + 3u + 1$

$= (-9u^3 - 3u^2) + (3u+1)$

$= -3u^2(3u+1) + (3u+1)$

Answer: $(1-3u^2)(3u+1)$

(h) $x^5 + x^4 + x + 1$

$= (x^5 + x^4) + (x+1)$

$= x^4(x+1) + (x+1)$

Answer: $(x^4+1)(x+1)$

(i) $y^3 - y^2 + y - 1$

$= (y^3 - y^2) + (y-1)$

$= y^2(y-1) + (y-1)$

Answer: $(y^2+1)(y-1)$

(j) $3s^3 + 5s^2 - 6s - 10$

$= (3s^3 + 5s^2) - (6s+10)$

$= s^2(3s+5) - 2(3s+5)$

Answer: $(s^2-2)(3s+5)$

6. Simplify the rational expression.

(a) $\frac{12x}{6x^2} = \frac{2 \cdot 6 \cdot x}{6 \cdot x \cdot x}$

Answer: $2/x$

(b) $\frac{5y^2}{10y + y^2} = \frac{5 \cdot y \cdot y}{y(10+y)}$

Answer: $\frac{5y}{(10+y)}$

(c) $\frac{3(x+2)(x-1)}{6(x-1)^2} = \frac{3(x+2)(x-1)}{2 \cdot 3(x-1)(x-1)}$

Answer: $\frac{x+2}{2(x-1)}$

(d) $\frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)}$

Answer: $\frac{1}{x+2}$

(e) $\frac{x^2+6x+8}{x^2+5x+4} = \frac{(x+2)(x+4)}{(x+1)(x+4)}$

Answer: $\frac{x+2}{x+1}$

(f) $\frac{y^2+y}{y^2-1} = \frac{y(y+1)}{(y-1)(y+1)}$

Answer: $\frac{y}{y-1}$

(g) $\frac{81x^3}{18x} = \frac{9 \cdot 9 \cdot x \cdot x^2}{2 \cdot 9 \cdot x}$

Answer: $\frac{9x^2}{2}$

(h) $\frac{14t^2 - t}{7t} = \frac{t(14t-1)}{7 \cdot t}$

Answer: $\frac{14t-1}{7}$

(i) $\frac{4(x^2-1)}{12(x-2)(x-1)} = \frac{4(x-1)(x+1)}{3 \cdot 4(x-2)(x-1)}$

Answer: $\frac{x+1}{3(x-2)}$

(j) $\frac{x^2-x-2}{x^2-1} = \frac{(x-2)(x-1)}{(x+1)(x-1)}$

Answer: $\frac{x-2}{x+1}$

Cont.

(k) $\frac{x^2+x-12}{x^2-5x+6} = \frac{(x+4)(x-3)}{(x-2)(x-3)}$

Answer: $\frac{x+4}{x-2}$

(l) $\frac{y^2-3y-18}{y^2+4y+3} = \frac{(y-6)(y+3)}{(y+1)(y+3)}$

Answer: $\frac{y-6}{y+1}$

7. Perform the multiplication or division and simplify.

(a) $\frac{4x}{x^2-4} \cdot \frac{x+2}{16x} = \frac{4x}{(x-2)(x+2)} \cdot \frac{(x+2)}{4 \cdot 4x}$

Answer: $\frac{1}{4(x-2)}$

(b) $\frac{x^2-2x-15}{x^2-9} \cdot \frac{x+3}{x-5} = \frac{(x-5)(x+3) \cdot (x+3)}{(x-3)(x+3)(x-5)}$

Answer: $\frac{x+3}{x-3}$

(c) $\frac{t-3}{t^2+9} \cdot \frac{t+3}{t^2-9} = \frac{(t-3)}{(t^2+9)} \cdot \frac{(t+3)}{(t-3)(t+3)}$

Answer: $\frac{1}{t^2+9}$

(d) $\frac{x^2-25}{x^2-16} \cdot \frac{x+4}{x+5} = \frac{(x-5)(x+5)(x+4)}{(x-4)(x+4)(x+5)}$

Answer: $\frac{x-5}{x-4}$

(e) $\frac{x^2+2x-3}{x^2-2x-3} \cdot \frac{3-x}{3+x} = \frac{(x+3)(x-1)(3-x)}{(x-3)(x+1)(3+x)}$

$= \frac{-(x-1)(x-3)}{(x-3)(x+1)}$

Answer: $\frac{-(x-1)}{x+1}$

(f) $\frac{x^2-x-6}{x^2+2x} \cdot \frac{x^3+x^2}{x^2-2x-3} = \frac{(x-3)(x+2) \cdot x^2(x+1)}{x(x+2) \cdot (x-3)(x+1)}$

Answer: x

(g) $\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15} = \frac{(x+3)}{(2x+3)(2x-3)} \cdot \frac{(2x-3)(x+5)}{(x+3)(x+4)}$

Answer: $\frac{x+5}{(2x+3)(x+4)}$

(h) $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$

$= \frac{(2x+1)}{(2x-5)(x+3)} \cdot \frac{(x+3)}{(3x-2)(2x+1)}$

Answer: $\frac{1}{(2x-5)(3x-2)}$

Cont.

(i) $\dfrac{2x^3+3x+1}{x^2+2x-15} \div \dfrac{x^2+6x+5}{2x^2-7x+3}$

$= \dfrac{(2x+1)(x+1)}{(x+5)(x-3)} \cdot \dfrac{(2x-1)(x-3)}{(x+1)(x+5)}$

Answer: $\dfrac{(2x+1)(2x-1)}{(x+5)(x+5)}$

(j) $\dfrac{4y^2-9}{2y^2+9y-18} \div \dfrac{2y^2+y-3}{y^2+5y-6}$

$= \dfrac{(2y-3)(2y+3)}{(2y-3)(y+6)} \cdot \dfrac{(y+6)(y-1)}{(2y+3)(y-1)}$

Answer: 1

8. Perform the addition or subtraction and simplify.

(a) $2+\dfrac{x}{x+3}$

$= \dfrac{2(x+3)}{x+3} + \dfrac{x}{x+3}$

Answer: $\dfrac{3(x+1)}{x+3}$

(b) $\dfrac{1}{x+5}+\dfrac{2}{x-3} = \dfrac{(x-3)+2(x+5)}{(x+5)(x-3)}$

Answer: $\dfrac{3x+7}{(x+5)(x-3)}$

(c) $\dfrac{1}{x+1}-\dfrac{1}{x+2} = \dfrac{(x+2)-(x+1)}{(x+1)(x+2)}$

Answer: $\dfrac{1}{(x+1)(x+2)}$

(d) $\dfrac{x}{(x+1)^2}+\dfrac{2}{x+1} = \dfrac{x}{(x+1)^2} + \dfrac{2(x+1)}{(x+1)^2}$

Answer: $\dfrac{3x+2}{(x+1)^2}$

(e) $\dfrac{1}{x^2}+\dfrac{1}{x^2+x} = \dfrac{1}{x \cdot x} + \dfrac{1}{x(x+1)}$

$= \dfrac{(x+1)+x}{x^2(x+1)}$

Answer: $\dfrac{2x+1}{x^2(x+1)}$

(f) $\dfrac{2x-1}{x+4}-1 = \dfrac{2x-1}{x+4} - \dfrac{(x+4)}{(x+4)}$

Answer: $\dfrac{x-5}{x+4}$

(g) $\dfrac{1}{x+1}+\dfrac{1}{x-1} = \dfrac{(x-1)+(x+1)}{(x+1)(x-1)}$

Answer: $\dfrac{2x}{(x+1)(x-1)}$

(h) $\dfrac{x}{x-4}-\dfrac{3}{x+6} = \dfrac{x(x+6)-3(x-4)}{(x-4)(x+6)}$

Answer: $\dfrac{x^2+3x+12}{(x-4)(x+6)}$

Cont.

(i) $\frac{5}{2x-3} - \frac{3}{(2x-3)^2} = \frac{5(2x-3)}{(2x-3)^2} - \frac{3}{(2x-3)^2}$

Answer: $\frac{2(5x-9)}{(2x-3)^2}$

(j) $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} = \frac{x^2}{x^3} + \frac{x}{x^3} + \frac{1}{x^3}$

Answer: $\frac{x^2+x+1}{x^3}$

9. Rationalise the denominator.

(a) $\frac{1}{2-\sqrt{3}} = \frac{(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

Answer: $2+\sqrt{3}$

(b) $\frac{2}{\sqrt{2}+\sqrt{7}} = \frac{2(\sqrt{2}-\sqrt{7})}{(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})}$

Answer: $\frac{2(\sqrt{7}-\sqrt{2})}{5}$

(c) $\frac{y}{\sqrt{3}+\sqrt{y}} = \frac{y(\sqrt{3}-\sqrt{y})}{(\sqrt{3}+\sqrt{y})(\sqrt{3}-\sqrt{y})}$

Answer: $\frac{y(\sqrt{3}-\sqrt{y})}{3-y}$

(d) $\frac{2}{3-\sqrt{5}} = \frac{2(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$

Answer: $\frac{3+\sqrt{5}}{2}$.

(e) $\frac{1}{\sqrt{x}+1} = \frac{\sqrt{x}-1}{(\sqrt{x}+1)(\sqrt{x}-1)}$

Answer: $\frac{\sqrt{x}-1}{x-1}$

(f) $\frac{2(x-y)}{\sqrt{x}-\sqrt{y}} = \frac{2(x-y)(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}$

Answer: $2(\sqrt{x}+\sqrt{y})$.

Cont.

10. Perform long division to find the quotient and remainder.

(a) $\frac{x^2 - 6x - 8}{x - 4}$

$$\begin{array}{r} x + 2 \\ x-4 \overline{) x^2 - 6x - 8} \\ -(x^2 - 8x) \\ \hline 2x - 8 \\ -(2x - 8) \\ \hline 0 \end{array}$$

Answer: $x+2$

(d) $\frac{x^3 + 3x^2 + 4x + 3}{3x + 6}$

$$\begin{array}{r} \frac{1}{3}x^2 + 1 \\ 3x+6 \overline{) x^3 + 3x^2 + 4x + 3} \\ -(x^3 \quad + 2x) \\ \hline 3x^2 + 2x + 3 \\ -(3x^2 \quad + 6) \\ \hline 2x - 3 \end{array}$$

Answer: $\frac{1}{3}x^2 + 1 + \frac{2x-3}{3x+6}$

(b) $\frac{4x^3 + 2x^2 - 2x - 3}{2x + 1}$

$$\begin{array}{r} 2x^2 - 1 \\ 2x+1 \overline{) 4x^3 + 2x^2 - 2x - 3} \\ -(4x^3 + 2x^2) \\ \hline -2x - 3 \\ -(-2x - 1) \\ \hline -2 \end{array}$$

Answer: $2x^2 - 1 - \frac{2}{2x+1}$

(e) $\frac{x^6 + x^4 + x^2 + 1}{x + 1}$

$$\begin{array}{r} x^5 - x^4 + 2x^3 - 2x^2 + 3x \\ x+1 \overline{) x^6 \quad + x^4 \quad + x^2 \quad + 1} \\ -(x^6 + x^5) \\ \hline -x^5 + x^4 \quad + x^2 \quad + 1 \\ -(-x^5 - x^4) \\ \hline 2x^4 \quad + x^2 \quad + 1 \\ -(2x^4 + 2x^3) \\ \hline -2x^3 + x^2 \quad + 1 \ \ldots \end{array}$$

Answer: $x^5 - x^4 + 2x^3 - 2x^2 + 3x - 3 + \frac{4}{x+1}$

(c) $\frac{x^3 - x^2 - 2x + 6}{x - 2}$

$$\begin{array}{r} x^2 + x \\ x-2 \overline{) x^3 - x^2 - 2x + 6} \\ -(x^3 - 2x^2) \\ \hline x^2 - 2x + 6 \\ -(x^2 - 2x) \\ \hline 6 \end{array}$$

Answer: $x^2 + x + \frac{6}{x-2}$

(f) $\frac{6x^3 + 2x^2 + 20x}{2x^2 + 5}$

$$\begin{array}{r} 3x + 1 \\ 2x^2+5 \overline{) 6x^3 + 2x^2 + 20x} \\ -(6x^3 \quad + 15x) \\ \hline 2x^2 + 5x \\ -(2x^2 \quad + 5) \\ \hline 5x - 5 \end{array}$$

Answer: $3x + 1 + \frac{5x-5}{2x^2-5}$.

The End.